Numerical-Perturbation Technique for Stability of Flat-Plate Boundary Layers with Suction

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A numerical-perturbation scheme is proposed for determining the stability of flows over plates with suction through a finite number of porous suction strips. The basic flow is calculated as the sum of the Blasius flow and closed-form linearized triple-deck solutions of the flow due to the strips. A perturbation technique is used to determine the increment a_{ij} in the complex wavenumber at a given location x_j due to the presence of a strip centered at x_i. The end result is a set of influence coefficients that can be used to determine the growth rates and amplification factors for any suction levels without repeating the calculations. The numerical-perturbation results are verified by comparison with interacting boundary layers for the case of six strips and the experimental data of Reynolds and Saric for single- and multiple-strip configurations. The influence coefficient form of the solution suggests a scheme for optimizing the strip configuration. The results show that one should concentrate the suction near branch I of the neutral stability curve, a conclusion verified by the experiments.

I. Introduction

SUCTION through porous strips is under consideration for laminar flow control. The effectiveness of this method—along with the optimal number, spacing, and mass flow rate through such strips—should be determined by stability calculations of the flow over a body with suction strips. For such calculations, the mean flow must first be determined as accurately as possible. Nayfeh and El-Hady[1] used a nonsimilar boundary-layer code to solve this problem. However, nonsimilar boundary-layer calculations fail to account for upstream influence. Methods that account for the upstream influence include numerical solutions of the Navier-Stokes equations, interacting boundary layers, and nonlinear triple-deck equations. In these solutions, one needs to calculate the mean flow at many streamwise locations (in fact, at many more than the number of locations where the stability calculations need to be performed) in order to determine accurate values for the basic state. After this, one needs to perform the stability calculations at many streamwise locations. We note that, if any change is made in the suction configuration, the mean flow as well as the stability characteristics need to be completely recalculated. Therefore, these solutions require prohibitively large amounts of computer time and storage; also, some of them may fail to converge for large Reynolds numbers. The purpose of the present paper is to devise a numerical-perturbation scheme that significantly reduces the computer resources required and hence can be used as a design tool.

The basic state is calculated as the sum of the Blasius flow and the triple-deck, closed-form solutions of Reed[2] and Nayfeh et al.[3] (which are slightly modified as described in Sec. III). Since these basic-state solutions are closed form, they do not require time-consuming marching and iteration. They are linear combinations of the Blasius flow and small corrections due to each porous strip; hence, they need not be completely recalculated if the suction configuration is changed. Instead of numerically solving the stability eigenvalue problem for the basic state, a perturbation scheme is used to calculate the eigenfunctions z_{nj} for the Blasius stability problem and their adjoints z^*_{nj} corresponding to the eigenvalue k_{Bj} at the streamwise location x_j. Then, the correction to k_{Bj} due to the presence of a suction strip of *unit* suction level centered at x_i is calculated in quadratures in terms of the z_{nj}, z^*_{nj}, k_{Bj}, and the closed-form, triple-deck solutions. The result is a set of influence coefficients a_{ij} that can be used to calculate the growth rates at the locations x_j due to the presence of a finite number of porous suction strips at x_i with the dimensional suction velocities $v^*_{\text{wall}_i}$. Integrating the growth rates results in a set of numbers c_i that can be used to calculate the amplification factor due to the presence of the strips as a linear combination. We should emphasize that the a_{ij} and c_i need not be recalculated if the suction levels are changed in the strips, provided the new levels remain compatible with the perturbation scheme. Moreover, a scheme is discussed in Sec. IV by which one need not completely recalculate the a_{ij} and c_i if the strip configuration is changed.

For the suction levels proposed for laminar flow control of wings,[4,5] the numerical-perturbation results are in agreement with the results of the interacting boundary layers for the case of six strips and the experimental results of Reynolds and Saric[6] for configurations with single and multiple porous strips. Since the amplification factor is a linear combination of the c_i, a scheme for optimizing the strip configuration is proposed. This scheme predicts that the suction needs to be concentrated near branch I of the neutral stability curve. This conclusion has also been verified by the experimental results of Reynolds and Saric.[6]

II. Stability Problem Formulation

Consider a steady, two-dimensional, incompressible boundary-layer flow over a flat plate at zero angle of attack with porous suction strips. We employ a Cartesian coordinate system such that x is in the streamwise direction along the plate and y is normal to the plate. To study the stability of such a

Received Sept. 14, 1982; revision received Feb. 18, 1985.

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N.B. This article is intended as a companion to "Experiments on the Stability of the Flat-Plate Boundary Layer with Suction," appearing on pages 202-207 of this issue.

basic state, or mean flow, we express each total flow quantity $\hat{q}$ as

$$\hat{q}(x,y,t) = Q_0(x,y) + q_1(x,y,t)$$

where $Q_0(x,y)$ is the basic-state component and $q_1(x,y,t)$ a small, unsteady disturbance. Substituting the total flow quantities $\hat{u}$, $\hat{v}$, and $\hat{p}$, the total streamwise velocity, normal velocity, and pressure, respectively, into the Navier-Stokes equations, subtracting the basic-state equations, and linearizing with the amplitude of the disturbance, we find, to first order, that the nondimensional disturbance equations are given by

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u_1}{\partial t} + U_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial U_0}{\partial x} + V_0\frac{\partial u_1}{\partial y} + v_1\frac{\partial U_0}{\partial y} = -\frac{\partial p_1}{\partial x} + \frac{1}{R}\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v_1}{\partial t} + U_0\frac{\partial v_1}{\partial x} + u_1\frac{\partial V_0}{\partial x} + V_0\frac{\partial v_1}{\partial y} + v_1\frac{\partial V_0}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{1}{R}\left(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2}\right) \tag{3}$$

Here, the Reynolds number is given by

$$R = U_\infty^* \delta / \nu_\infty^*$$

where δ is a reference boundary-layer thickness defined as

$$\delta = \sqrt{\nu_\infty^* x^* / U_\infty^*}$$

and ν_∞^*, x^*, and U_∞^* are the dimensional freestream kinematic viscosity, streamwise position along the plate, and freestream velocity, respectively.

The boundary conditions for the disturbance equations (1-3) are

$$u_1 = v_1 = 0 \text{ at } y=0, \quad u_1 \text{ and } v_1 \to 0 \text{ as } y \to \infty \tag{4}$$

These were shown to be reasonable approximations by Lekoudis[7] under certain conditions. At the wall, the streamwise velocity component u_1 of the disturbance is taken to be zero, provided that the percentage of the permeable area is small and most of the flow there is directed normal to the surface. Lekoudis investigated the effect of the wall admittance by letting

$$v_1/p_1 = f \text{ at } y=0 \tag{5}$$

where f is the wall admittance. For closely spaced perforations and small surface permeability, he found that $v_1(x,0,t) = 0$ is an acceptable approximation. When the suction velocity is large, this conditions breaks down.

We expand u_1, v_1, and p_1 in the form of traveling, harmonic Tollmien-Schlichting waves as

$$q_1 = q(y)\exp(i\theta) \tag{6}$$

where the phase function θ is defined by

$$\frac{\partial \theta}{\partial x} = k, \quad \frac{\partial \theta}{\partial t} = -\omega \tag{7}$$

The quantities k and ω are the dimensionless streamwise wavenumber and frequency, respectively, defined by

$$k = k^*\delta, \quad \omega = \omega^*\delta/U_\infty^*$$

We consider the case of spatial stability, so that the wavenumber k is complex and the frequency ω is real. Since the Reynolds numbers are large, nonparallel effects are negligible.

Substituting Eqs. (6) and (7) into Eqs. (1-4) and considering only mean flows of the form,

$$U_0 = U_0(y) + \mathcal{O}(1/R), \quad V_0 = \hat{V}_0(y)/R$$

$$\hat{V}_0 = \mathcal{O}(1), \quad P_0 = \text{const}$$

we find that

$$iku + \frac{\mathrm{d}v}{\mathrm{d}y} = 0 \tag{8}$$

$$i(kU_0 - \omega)u + v\frac{\mathrm{d}U_0}{\mathrm{d}y} = -ikp + \frac{1}{R}\left(\frac{\mathrm{d}^2 u}{\mathrm{d}y^2} - k^2 u\right) \tag{9}$$

$$i(kU_0 - \omega)v = -\frac{\mathrm{d}p}{\mathrm{d}y} + \frac{1}{R}\left(\frac{\mathrm{d}^2 v}{\mathrm{d}y^2} - k^2 v\right) \tag{10}$$

$$u = v = 0 \text{ at } y=0, \quad u \text{ and } v \to 0 \text{ as } y \to \infty \tag{11}$$

The system of Eqs. (8-11) represents an eigenvalue problem for the parameters k, ω, and R. For known basic-state velocity profiles, the equations are integrated numerically using a computer code developed by Scott and Watts[8] to handle stiff two-point boundary value problems such as this. Specifying R and ω, we find the eigenvalue

$$k = k_r + ik_i$$

where k_r and k_i are the real and imaginary parts of k, respectively. Then $-k_i$ is the spatial growth rate of the disturbance.

From k_i, we determine the amplification factor as

$$n = \ell n \frac{A}{A_0} = -2\int_{R_0}^{R} k_i(R)\mathrm{d}R \tag{12}$$

where R_0 is the square root of the x Reynolds number where the constant-frequency disturbance first becomes unstable and A and A_0 are the amplitudes of the disturbance at x^* and x_0^*, respectively. Although the e^n method[9-11] cannot be used to predict the exact location of transition because of the strong dependence of transition location on the freestream turbulence levels and because it ignores the nonlinear breakdown to turbulence, it is useful as a design tool because it predicts trends for those changes in the mean flow that delay transition.

III. Basic State—Linearized Triple Deck

Laminar viscous flow over a flat plate with a discrete modulation exhibits a triple-deck structure[12-14] (see also Refs. 15-18). In this case, the modulation is caused by a porous strip centered at a dimensional distance x_p^* far from the plate's leading edge $x^* = 0$. Upstream of the region influenced by the suction is the Prandtl boundary layer. The flow in the neighborhood of the porous strip is described by three decks or nested boundary layers. The small parameter in this problem is $\hat{\epsilon}$, which is defined as

$$\hat{\epsilon} = Re^{-1/8}, \quad \text{where } Re = x_p^* U_\infty^* / \nu_\infty^*$$

According to triple-deck theory, the streamwise effects of the modulation occur in a neighborhood $\mathcal{O}(\hat{\epsilon}^3 x_p^*)$ of x_p^*. The middle deck, the displaced Prandtl layer, is $\mathcal{O}(\hat{\epsilon}^4 x_p^*)$ thick and characterized by rotational, inviscid modulations. The upper deck, whose thickness is $\mathcal{O}(\hat{\epsilon}^3 x_p^*)$, has inviscid, irrotational modulations. The lower deck is $\mathcal{O}(\hat{\epsilon}^5 x_p^*)$ thick and has viscous, rotational modulations. The wall boundary conditions are satisfied by the lower-deck governing equations. The linearized triple-deck equations we derive are expected to be valid only for strip widths $\mathcal{O}(\hat{\epsilon}^3 x_p^*)$ and suction levels $\mathcal{O}(\hat{\epsilon}^3 U_\infty^*)$.

For the case of injection through one porous strip, Napolitano and Ressick[19] developed a closed-form solution for the linearized lower-deck equations by means of Fourier transforms and calculated only the wall pressure coefficient and shear stress. Following their work, Reed[2] and Nayfeh et al.[3] obtained expressions for the pressure and shear, as well as the velocity components and displacement thickness for one finite-length suction strip. Then they developed closed-form solutions valid at all points in the direction normal to the plate. Finally, noting the linearity of the problem and using superposition, they developed closed-form solutions valid for any number of suction strips.

In this paper, we examine the stability of the basic state given by slightly modified versions of these linearized triple-deck, closed-form solutions. Here, we consider N porous strips, centered at $x_1^*, x_2^*, \ldots, x_N^*$, with the leading and trailing edges $x_{\mathrm{LE}_1}^*, x_{\mathrm{LE}_2}^*, \ldots, x_{\mathrm{LE}_N}^*$ and $x_{\mathrm{TE}_1}^*, x_{\mathrm{TE}_2}^*, \ldots, x_{\mathrm{TE}_N}^*$, respectively. All are dimensional distances from the plate's leading edge. We define the x Reynolds number at x^* and the ith strip, respectively, as

$$Re_x = x^* U_\infty^* / \nu_\infty^*, \quad Re_{x_i} = x_i^* U_\infty^* / \nu_\infty^* \tag{13}$$

Using these definitions, we express the streamwise component of velocity as[2,3]

$$\frac{u^*}{U_\infty^*}(x^*,y^*) = f'(\eta) + \sum_{i=1}^{N} Re_{x_i}^{1/4} \lambda^{-1/2} \frac{v_{\mathrm{wall}_i}^*}{U_\infty^*} \times \left[\frac{f''(\eta)}{\lambda\sqrt{2}} - 1\right] \bar{\delta}(x-x_i) + \bar{u}(x-x_i,y) \tag{14}$$

where

$$\bar{u}(x-x_i,y) = \bar{u}_\infty \left[\frac{\lambda^{5/4}}{x_i^*}(x^* Re_x^{3/8} - x_{\mathrm{LE}_i}^* Re_{x_i}^{3/8}),\ \frac{Re_{x_i}^{5/8}}{x^*} \lambda^{3/4} y^*\right] - \bar{u}_\infty \left[\frac{\lambda^{5/4}}{x_i^*}(x^* Re_x^{3/8} - x_{\mathrm{TE}_i}^* Re_{x_i}^{3/8}),\ \frac{Re_{x_i}^{5/8}}{x^*} \lambda^{3/4} y^*\right] \tag{15}$$

$$\bar{\delta}(x-x_i) = \bar{\delta}_\infty \left[\frac{\lambda^{5/4}}{x_i^*}(x^* Re_x^{3/8} - x_{\mathrm{LE}_i}^* Re_{x_i}^{3/8})\right] - \bar{\delta}_\infty \left[\frac{\lambda^{5/4}}{x_i^*}(x^* Re_x^{3/8} - x_{\mathrm{TE}_i}^* Re_{x_i}^{3/8})\right] \tag{16}$$

$$\begin{aligned}\bar{u}_\infty(x,y) = &-\frac{3}{2\pi i} \frac{|x|^{2/3}}{\theta^{4/3}} \left[\int_0^{\infty e^{i\pi}} \frac{e^\tau}{\tau^{5/3}} \int_0^{(\tau^{1/3} y)/|x|^{1/3}} A_i(\eta)\,\mathrm{d}\eta\,\mathrm{d}\tau + \int_{\infty e^{-i\pi}}^{0} \frac{e^\tau}{\tau^{5/3}} \int_0^{(\tau^{1/3} y)/|x|^{1/3}} A_i(\eta)\,\mathrm{d}\eta\,\mathrm{d}\tau\right] \\ &+ \frac{9}{2\pi\theta^2} \int_0^\infty \frac{e^{-i7\pi/6}\rho}{\rho^4 + e^{i7\pi/6}} \int_0^{e^{i\pi/3}\rho\theta^{1/3} y} A_i(\eta)\,\mathrm{d}\eta\, e^{-\rho^3\theta|x|}\,\mathrm{d}\rho \\ &+ \frac{9}{2\pi\theta^2} \int_0^\infty \frac{e^{i7\pi/6}\rho}{\rho^4 + e^{i7\pi/6}} \int_0^{e^{-i\pi/3}\rho\theta^{1/3} y} A_i(\eta)\,\mathrm{d}\eta\, e^{-\rho^3\theta|x|}\,\mathrm{d}\rho, \quad \text{for } x>0\end{aligned} \tag{17a}$$

$$\bar{u}_\infty(x,y) = -\frac{9}{\pi\theta^2} \int_0^\infty \frac{\rho}{\rho^8+1} \int_0^{\rho\theta^{1/3} y} A_i(\eta)\,\mathrm{d}\eta\, e^{-\rho^3\theta|x|}\,\mathrm{d}\rho, \quad \text{for } x<0 \tag{17b}$$

$$\bar{\delta}_\infty(x) = -\frac{|x|^{2/3}}{\theta^{4/3}\Gamma(5/3)} - \frac{3}{2\pi\theta^2} \int_0^\infty \frac{\sqrt{3}\rho^5 - \rho}{\rho^8 - \sqrt{3}\rho^4 + 1} e^{-\rho^3\theta|x|}\,\mathrm{d}\rho, \quad \text{for } x>0 \tag{18a}$$

$$\bar{\delta}_\infty(x) = -\frac{3}{\pi\theta^2} \int_0^\infty \frac{\rho}{\rho^8+1} e^{-\rho^3\theta|x|}\,\mathrm{d}\rho, \quad \text{for } x<0 \tag{18b}$$

Here, the quantities with an asterisk are dimensional. The function $f(\eta)$ in Eq. (14) is the Blasius function.[20] The constant λ is defined as

$$\lambda = f''(0)/\sqrt{2} \tag{19}$$

Each individual strip solution in Eq. (14) is valid in the immediate neighborhood of the corresponding strip, but neglects streamtube divergence. In the case of multiple modulations along a flat plate, to account for the streamtube divergence, we scale x (and not x_i) in the arguments of $\bar{\delta}$ and $\bar{u}$ with the x Reynolds number at x^* and scale y with x^*. This improves the accuracy of the superposed solutions in comparison with the proposed solutions of Refs. 2 and 3.

IV. Perturbation Scheme

Taking advantage of the linearity of the triple-deck formulation, we develop a perturbation scheme that can be used to predict efficient strip configurations. The technique is used to determine the increment in the complex wavenumber at a given location due to the presence of a strip elsewhere.

We consider the first-order system of disturbance equations as

$$iku + Dv = 0 \tag{20}$$

$$-i\omega u + ikuU + vDU = -ikp + (1/R)(D^2u - k^2u) \tag{21}$$

$$-i\omega v + ikvU = -Dp + (1/R)(D^2v - k^2v) \tag{22}$$

where $D \equiv \mathrm{d}/\mathrm{d}y$. We take the mean flow U as

$$U = U_0 + \epsilon U_1, \quad |\epsilon| \ll 1, \quad U_1 = \mathcal{O}(1) \tag{23}$$

This mean flow is exactly as in the triple-deck expression for u^*/U_∞ where

$$U_0 = f'(\eta)\text{(Blasius)}, \quad \epsilon = \frac{v_{\mathrm{wall}_i}^*}{U_\infty^*} Re_{x_i}^{1/4} \lambda^{-1/2} \tag{24}$$

Then we expand

$$u = u_0 + \epsilon u_1 + \ldots, \quad v = v_0 + \epsilon v_1 + \ldots \tag{25a}$$

$$p = p_0 + \epsilon p_1 + \ldots, \quad k = k_0 + \epsilon k_1 + \ldots \tag{25b}$$

where the quantities with subscript 0 represent Blasius disturbance quantities and those with subscript 1 represent the contributions due to the presence of the suction strips. The zeroth-order problem is

$$L_1(u_0,v_0,p_0) = ik_0u_0 + Dv_0 = 0 \tag{26a}$$

$$L_2(u_0,v_0,p_0) = -i\omega u_0 + ik_0u_0U_0 + v_0DU_0 + ik_0p_0 - (1/R)(D^2u_0 - k_0^2u_0) = 0 \tag{26b}$$

$$L_3(u_0,v_0,p_0) = -i\omega v_0 + ik_0v_0U_0 + Dp_0 - (1/R)(D^2v_0 - k_0^2v_0) = 0 \tag{26c}$$

$$u_0, v_0 = 0 \text{ at } y=0 \text{ and } u_0, v_0 \to 0 \text{ as } y \to \infty \tag{26d}$$

The system of Eqs. (26) defines an eigenvalue problem for the Blasius boundary-layer flow.

The first-order problem is

$$L_1(u_1,v_1,p_1) = -ik_1u_0 \tag{27a}$$

$$L_2(u_1,v_1,p_1) = -ik_1u_0U_0 - ik_0u_0U_1 - v_0DU_1 - ik_1p_0 + (1/R)(-2k_0k_1u_0) \tag{27b}$$

$$L_3(u_1,v_1,p_1) = -ik_1v_0U_0 - ik_0v_0U_1 + (1/R)(-2k_0k_1v_0) \tag{27c}$$

$$u_1,v_1 = 0 \text{ at } y=0 \text{ and } u_1,v_1 \to 0 \text{ as } y\to\infty \tag{27d}$$

A solution exists for this system [Eqs. (27)] only if a solvability condition is satisfied.[21] The inhomogeneous terms on the right-hand sides must be orthogonal to every solution of the adjoint homogeneous problem. That is,

$$k_1\int_0^\infty \left[(2k_0 + iRU_0)z_2^*z_1 + iRz_2^*z_4 - iz_3^*z_1 - \frac{i}{R}z_4^*z_2 - \frac{2k_0}{R}z_4^*z_3 - iU_0z_4^*z_3\right]\mathrm{d}y + \int_0^\infty (iRk_0U_1z_2^*z_1 + RDU_1z_2^*z_3 - ik_0U_1z_4^*z_3)\mathrm{d}y = 0 \tag{28a}$$

or

$$k_1\int_0^\infty f(x,y)\mathrm{d}y + \int_0^\infty g(x,y)\mathrm{d}y = 0 \tag{28b}$$

where

$$z_1 = u_0, \quad z_2 = Du_0, \quad z_3 = v_0, \quad z_4 = p_0$$

and the starred quantities are the corresponding adjoint solutions. It follows from Eq. (28b) that

$$k_1 = -\int_0^\infty g(x,y)\mathrm{d}y \Big/ \int_0^\infty f(x,y)\mathrm{d}y \tag{29}$$

At each x, since k_1 is independent of ϵ, the correction to k_0 is a linear function of ϵ. Recalling that ϵ is given by Eq. (24), we conclude that ϵk_1 is directly proportional to the suction rate.

Using this property of linearity, we consider a plate with N porous strips and M specified points of computation between branches I and II of the stability curve. The zeroth-order eigenvalue problem is solved to find the Blasius complex wavenumber k_{B_j} and its corresponding eigenvectors $z_{\ell j}$ and $z_{\ell j}^*$, $\ell = 1,\ldots,4$ at each point $j = 1,\ldots,M$. Then, using linear triple-deck theory and considering Eqs. (14), (28), and (29), the y-dependent function is calculated to be

$$\left[\frac{f''(\eta)}{\lambda\sqrt{2}} - 1\right]\delta(x_j - x_i) + \bar{u}(x_j - x_i, y) \tag{30}$$

at each point of computation x_j, $j = 1,\ldots,M$ due to an individual strip located at x_i, $i = 1,\ldots,N$. Then, applying Eqs. (28) and (29), we determine k_1 for each i and j and call this quantity a_{ij}. Thus, the total complex wavenumber k_j at point x_j due to the presence of all N strips is

$$k_j = k_{B_j} + \sum_{i=1}^N a_{ij}\frac{v^*_{\text{wall}_i}}{U^*_\infty}Re_{x_i}^{1/4}\lambda^{-1/2} \tag{31}$$

Using the trapezoidal rule to perform the integration in Eq. (12) and applying Eq. (31), we find

$$n = \ell n\frac{A}{A_0} = -\text{Imag}\left[\sum_{j=2}^M (k_{B_j} + k_{B_{j-1}})(R_j - R_{j-1}) + \sum_{j=2}^M\sum_{i=1}^N (a_{ij} + a_{ij-1})\frac{v^*_{\text{wall}_i}}{U^*_\infty}Re_{x_i}^{1/4}\lambda^{-1/2}(R_j - R_{j-1})\right] \tag{32}$$

or

$$n = n_B + \sum_{i=1}^N c_i\frac{v^*_{\text{wall}_i}}{U^*_\infty} \tag{33}$$

where

$$c_i = -\text{Imag}\sum_{j=2}^M (a_{ij} + a_{ij-1})Re_{x_i}^{1/4}(R_j - R_{j-1})\lambda^{-1/2} \tag{34}$$

and n_B is the Blasius amplification factor. We emphasize that the a_{ij} are independent of the strip suction levels $v^*_{\text{wall}_i}$ and therefore need not be recalculated if the suction levels are changed in the strips. One can compute the a_{ij} for any number of strips, the limit of which is, of course, to consider strips edge-to-edge all the way from branch I to branch II; in the end, one can specify zero suction levels for those strips not needed. With this method, we can vary the number of, spacing between, and mass flow rate through the strips without recalculation. Moreover, with the limitation of being able to consider only multiples of the original widths, we can vary the strip width by keeping neighboring strips and considering them as one.

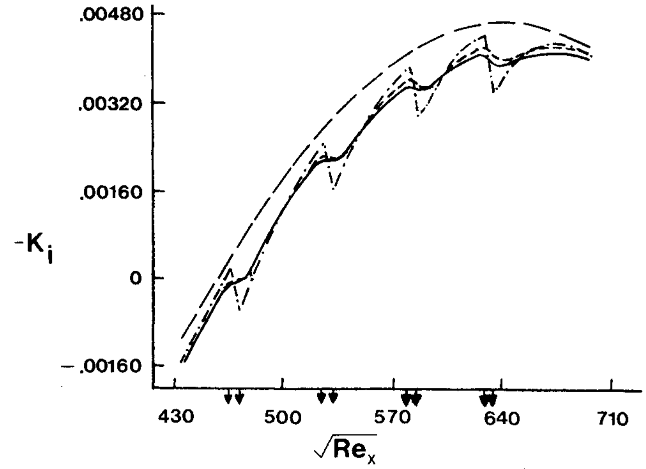

Fig. 1 Overall view of the spatial growth rate $-k_i$ vs $\sqrt{Re_x}$ for the third through sixth strips ($v^*_{\text{wall}} = -2.3\times10^{-4}$ U^*_∞, $F = 86\times10^{-6}$: —, linear triple deck; ---, interacting boundary layers; -·-, nonsimilar boundary layer; ——, Blasius flow.

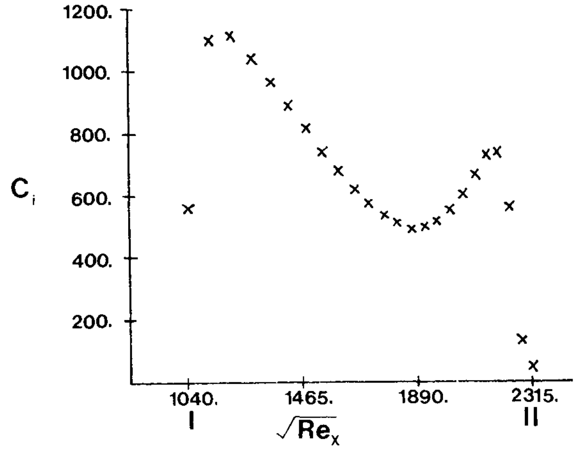

Fig. 2 Influence coefficients c_i for 1.6 cm wide strips and a dimensionless frequency $F = 20\times10^{-6}$.

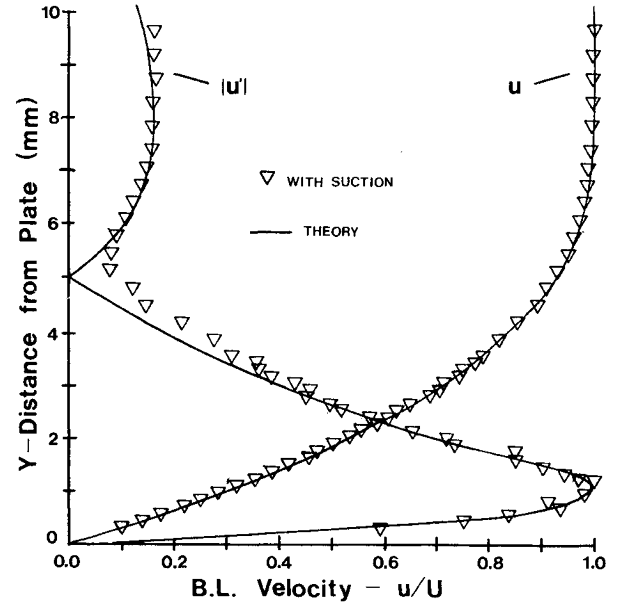

Fig. 3 Comparison of theory with experiment: one suction strip located at $x=194.3$ cm is open with a suction level of $v^*_{\rm wall}=-5.7\times10^{-3}$ U^*_∞ (dimensionless frequency is $F=25\times10^{-6}$ and square root of the unit Reynolds number per meter is $R_u=961$ m$^{-1/2}$, strip is 1.6 cm wide, and measurements are taken at $x=205$ cm): ∇, suction, experiment[6]; —, corresponding theory.

V. Results and Discussion

First, we present growth rate comparisons for the linearized triple deck, the interacting boundary layer (described briefly in the Appendix), and the nonsimilar boundary-layer solutions (also described in the Appendix) for the case of a flat plate with six strips each of width 0.02 m centered at 0.18 m intervals. The first strip is centered at a distance of 0.3 m from the plate's leading edge. The suction velocity through each strip is -2.3×10^{-4} U_∞ and Re_x at the center of the first strip is 1×10^5. Superimposed onto this basic state is a disturbance with the constant dimensionless frequency $F=2\pi\nu^*_\infty f/U^{*2}_\infty=86\times10^{-6}$ (f is the dimensional frequency in hertz). Figure 1 shows the spatial growth rate $-k_i$ plotted vs $R=\sqrt{Re_x}$ in an overall picture of the third through sixth strips. The disturbance upstream of the third strip is stable and therefore not shown. It is apparent from Fig. 1 that the comparisons between the interacting boundary-layer solution and the superposed, linearized, triple-deck solutions are good.

By comparing the linearized triple-deck solutions and the interacting boundary-layer solutions with the undisturbed, or Blasius, solutions, we find that even for such small suction levels the growth rates are dramatically decreased in the neighborhoods of the strips, thus stabilizing the flow. There is a relatively small upstream influence of the suction, its extent being the order of a couple of boundary-layer thicknesses; also, it appears to decay quickly. That is, the region upstream of the strip affected by the suction is short. In contrast, there is a larger downstream influence and it lingers for quite a distance down the plate, as much as $\mathcal{O}(20\delta)$. In comparison, the nonsimilar boundary-layer calculations are poor, particularly in the neighborhood of a strip. They predict no upstream influence and the chordwise gradients of the quantities are discontinuous at the leading and trailing edges of the strip. This is due to the parabolic nature of the equations and the discontinuity in the wall-suction boundary conditions as one marches along the plate. Smoothing occurs when interaction with the potential flow solution, as in interacting boundary layers, is taken into account.

The influence coefficient form of the perturbation solution [Eq. (33)] suggests a scheme for optimizing the strip configuration. To obtain an efficient strip distribution, the amplification factor n is minimized while maintaining a constant total mass flow rate. This requires

$$\min\sum_{i=1}^{N} c_i\frac{v^*_{{\rm wall}_i}}{U^*_\infty} \tag{35}$$

Since the c_i are independent of the suction levels, Eq. (35) reduces to a linear minimization problem for the solution $v^*_{{\rm wall}_i}/U^*_\infty$, $i=1,\ldots,N$. For each strip $i=1,\ldots,N$, the coefficient c_i of $v^*_{{\rm wall}_i}/U^*_\infty$ is evaluated and stored. Since the minimization problem is linear and since $v^*_{{\rm wall}_i}/U^*_\infty$ is negative, the largest c_i correspond to the strips where suction should be concentrated for optimization. A typical c_i distribution is shown in Fig. 2. Since the largest coefficients occur near branch I, the distribution indicates that suction should be concentrated near branch I of the stability curve for efficiency. Disturbances must be controlled while their growth rates are still small.

We now compare our numerical-perturbation scheme results with the experiments of Reynolds and Saric.[6] [The testing involved a 4 m chord by 1.83 m span by 0.02 m thick flat-plate model with Dynapore porous suction panels located at the four possible positions shown in Fig. 1 of Ref. 6. (All distances are in centimeters from the leading edge.) Characteristics of the boundary layer with and without suction were determined with a constant-temperature hot wire.]

Typical profiles measured 11 cm downstream of a single 1.6 cm strip at $x=194.3$ cm in a flow of square root unit Reynolds number $Ru=U^*_\infty/\nu^*_\infty=961$ m$^{-1/2}$ are shown in Fig. 3. The suction velocity is -5.7×10^{-3} U^*_∞ and the dimensionless disturbance frequency is $F=25\times10^{-6}$. Since the linear stability theory cannot predict the actual levels of the disturbances, the theoretical disturbance profile was normalized using the experimentally obtained maximum disturbance amplitude (0.1%). Figure 3 shows that the theoretical mean velocity as well as the disturbance velocity profiles are in good agreement with the experimental results, giving us confidence in the triple-deck model.

Figures 7 and 8 of Ref. 6 and Figs. 4 and 5 of this paper compare the theoretically predicted integrated disturbance profiles (i.e., e^n) with the experimental results of Reynolds and Saric[6] for single- and multiple-strip configurations. In these figures, the theoretical amplitudes have been normalized to the initial or lowest Reynolds number measurement. In all figures, the top two curves correspond to the no-suction or Blasius flow; they are included for reference and because they indicate the degree of smoothness of the model with the suction panels in place. Consequently, the deviations between the experimental and theoretical results for the no-suction case indicate the type of agreement expected for the case of suction.

In the single 1.6 cm width strip cases, Fig. 7 of Ref. 6 and Fig. 4 of this paper, the square root unit Reynolds numbers are 961 and 987 m$^{-1/2}$, respectively, and the dimensionless disturbance frequencies are 25 and 20×10^{-6}, respectively. In spite of the intense suction level ($v^*_{\rm wall}=-5.7$ and -5.5 $\times10^{-3}$ U^*_∞, respectively), the agreement between experiment and theory is fairly good, with the theoretical curves following the undulations in the experimental data at and around the strips. The theory predicts the upstream influence of the strips rather closely. The predicted downstream influence extends to as much as 20 boundary-layer thicknesses, in agreement with the experimental data. However, the theory predicts a larger downstream influence than that observed. This discrepancy may be due to the fairly intense suction level associated with a single strip and the consequent beginning of breakdown in the theory; the theory is expected to be valid up to suction levels $\mathcal{O}(4\times10^{-3})$ in this case. This applies to growth rates. Subsequent exponentiation of the n factor (cumulative growth rate) to get the amplitude magnifies otherwise small errors. We note that, in comparison with the no-suction, or Blasius, amplitudes, the differences between theory and experiment for the suction cases are relatively small.

In light of the prediction of the optimization scheme that suction should be placed close to branch I, Reynolds and Saric[6] evaluated different strip locations for a single strip under otherwise fixed conditions: $F=20\times10^{-6}$, $v^*_{\text{wall}} = -5.5\times10^{-3}\ U^*_\infty$, and $Ru=987\ \text{m}^{-1/2}$. Figure 4 shows two strip locations, 194.3 and 247.6 cm from the plate's leading edge; the branch I neutral point occurs at 110.5 cm. To evaluate the effect of strip location, one needs to compare the final disturbance amplitude at some downstream location of a given initial disturbance. Thus, the experimental results were normalized at a location far upstream of the forward strip so that the initial amplitudes coincide there. The theoretical results were also normalized there. Figure 4 clearly shows the advantage of placing the suction strip forward, closer to branch I of the neutral-stability curve.

The multiple strip case is more representative of LFC systems; typical configurations are shown in Fig. 8 of Ref. 6 and Fig. 5 of this paper for strip widths of 1.6 cm and a disturbance frequency of $F=25\times10^{-6}$. The total mass flow rates are the same as for the single-strip results. Figure 8 of

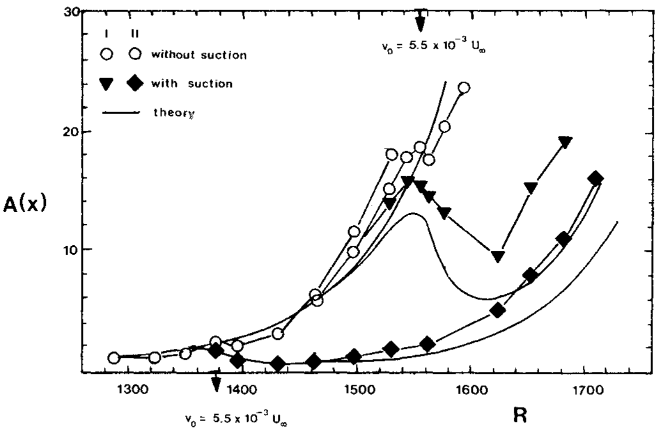

Fig. 4 Effect of moving a single strip upstream toward branch I of the neutral stability curve ($F=20\times10^{-6}$, $R_u=987\ \text{m}^{-1/2}$, and $v^*_{\text{wall}}=-5.5\times10^{-3}\ U^*_\infty$): ○, no suction, experiment[6]; ◆, forward suction strip location, experiment[6]; ▼, aft suction strip location, experiment[6]; —, corresponding theory.

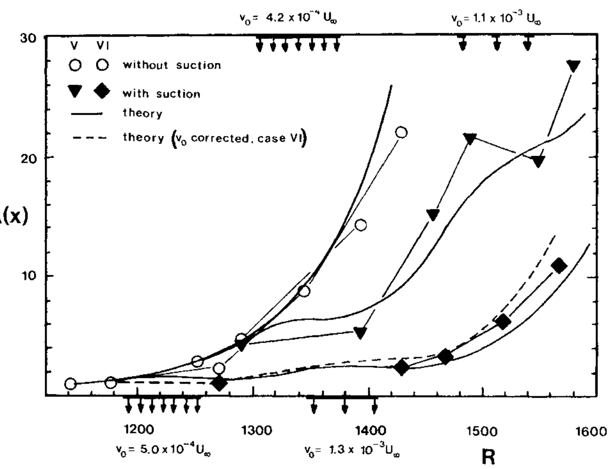

Fig. 5 Effect of moving multiple strips upstream toward branch I of the neutral-stability curve: ○, no suction, experiment[6]; ▼, suction, experiment[6] (indicated on upper scale); ◆, suction moved upstream toward branch I, experiment[6] (indicated on lower scale); —, corresponding theory; ---, adjusted theory.

Ref. 6 shows a six-strip configuration, three on each of the first and second panels. The suction level in each strip is $v^*_{\text{wall}} = -1.0\times10^{-3}\ U^*_\infty$ and the square root unit Reynolds number is $Ru=961\ \text{m}^{-1/2}$. Figure 5 shows results for a ten-strip configuration, seven on the first panel and three on the second. The upper curve corresponds to a square root unit Reynolds number of $Ru=961\ \text{m}^{-1/2}$ and suction levels on the first and second panels of $v^*_{\text{wall}} = -4.2\times10^{-4}$ and -1.1×10^{-3}, respectively, while the lower curve corresponds to $Ru=877\ \text{m}^{-1/2}$ and $v^*_{\text{wall}} = -5.0\times10^{-4}$ and -1.3×10^{-3}, respectively. Since the suction levels are much less than those for single-strip configurations, the upstream influence ahead of the first strip is reduced. The downstream influence is a cumulative effect. Also, we note that with smaller individual suction levels, the agreement between theory and experiment is very good in the seven/three case.

For the ten-strip configuration, Reynolds and Saric[6] performed two experiments, the first at $Ru=961\ \text{m}^{-1/2}$ and the second at $Ru=877\ \text{m}^{-1/2}$. By lowering the unit Reynolds number, the suction strips are effectively moved forward toward branch I of the neutral-stability curve. The relative locations of the strips are shown on the upper and lower horizontal axes of Fig. 5. The experimental and theoretical disturbance amplitudes were normalized to coincide at a location far ahead of the first suction strip. Figure 5 shows a good agreement between the theoretical and experimental results and that a 55% reduction in the disturbance amplitude is achieved by moving the suction forward near branch I of the neutral-stability curve. We note that lowering Ru from 961 to 877 $\text{m}^{-1/2}$ resulted in a slight increase in the value of v^*_{wall}. Using the corrected value, we recalculated the disturbance amplitude; it is shown as the dashed curve in Fig. 5. The two theoretical curves bound the experimental results.

VI. Conclusions

In this paper we propose a numerical-perturbation scheme for determining the stability of flows over plates with suction through a finite number of porous suction strips; in the companion paper of Reynolds and Saric, the theory is successfully verified by experiment. The results of the theory provide accurate linear, closed-form solutions for the basic flow that account for upstream influence, a set of influence coefficients that can be used to determine the growth rates and amplification factors for any suction levels without repeating the calculations, and a simple linear optimization scheme that correctly predicts that suction should be concentrated near branch I of the neutral stability curve, i.e., when disturbances are still small in amplitude. The numerical-perturbation technique we propose is reliable enough to replace the experiment as a tool in designing efficient strip configurations insofar as two-dimensional, incompressible flows are concerned.

Appendix: Interacting and Nonsimilar Boundary-Layer Solutions

Following the work of Ragab and Nayfeh,[22] we introduce into the dimensionless boundary-layer equations the Levy-Lees variables

$$\xi=\int_0^x U_e(x)\mathrm{d}x,\quad \eta=\frac{U_e(x)y\sqrt{Re_x}}{\sqrt{2\xi}}$$

$$F(\xi,\eta)=\frac{u}{U_e(x)},\quad V(\xi,\eta)=\frac{2\xi}{U_e(x)}\left(F\frac{\partial\eta}{\partial x}+\frac{v\sqrt{Re_x}}{\sqrt{2\xi}}\right)$$

where $U_e(x)$ is the local chordwise edge speed. The resulting system of equations is

$$2\xi F_\xi + V_\eta + F = 0$$

$$2\xi FF_\xi + VF_\eta + \beta(F^2 - 1) - F_{\eta\eta} = 0$$

$$\beta = -\frac{2\xi}{U_e^3}\frac{\mathrm{d}p}{\mathrm{d}x}$$

The corresponding boundary conditions are

$$F = 0 \text{ at } \eta = 0, \quad F \to 1 \text{ as } \eta \to \infty$$

$$F = F_0(\eta) \text{ at } \xi = \xi_0 \begin{array}{c}\text{far}\\ \text{upstream}\end{array}$$

$$V = v_{\text{wall}} \frac{\sqrt{2\xi Re_x}}{U_e(x)} \xi_{\text{strip LE}} \leq \xi \leq \xi_{\text{strip TE}} \text{ at } \eta = 0$$

$$= 0 \quad \text{otherwise}$$

Specifying the pressure distribution, these equations are solved using a second-order accurate, finite difference marching scheme. A Newton-Raphson procedure is used to quasilinearize the nonlinear terms, giving the momentum and continuity equations coupled in their linearized form, the so-called Davis coupled scheme.[23] The viscous displacement thickness

$$\delta = \frac{\sqrt{2\xi}}{U_e(x)\sqrt{Re_x}} \int_0^\infty (1 - F)\,\mathrm{d}\eta$$

is iteratively made to equal the inviscid displacement thickness (i.e., the displacement thickness from boundary-layer interaction with the inviscid flow) given by

$$\frac{\mathrm{d}y_D}{\mathrm{d}x} = \frac{1}{\pi} \int_{\text{LE}}^\infty \frac{p(t)}{x - t}\,\mathrm{d}t$$

To obtain the nonsimilar solution, we set $\beta = 0$ in the above equations and numerically integrate once along the plate.

Acknowledgments

The authors are indebted to Drs. William Saric and Gregory Reynolds for their helpful comments and suggestions. The authors are also grateful to the reviewers who carefully read and made suggestions to improve this paper and its companion by Reynolds and Saric. This work was funded under NASA Grant NSG-1255 and Office of Naval Research Contract N00014-75-C-0381.

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